

Dynamic Ownership, Private Benefits, and Stock Prices

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Abstract

I quantify private benefits of control, and their impact on stock prices, by estimating a structural model of optimal shareholding using data on the ownership dynamics of Italian public companies. The results show that controlling shareholders generally have positive and persistent impact on stock prices, and the impact is larger during the last Eurozone debt crisis. The results imply that controlling shareholders are particularly beneficial for the rest of the company shareholders during negative economic cycles. I also provide evidence of a synergistic effect when the controlling shareholder is a corporation.

Keywords: Private Benefits, Controlling Shareholders, Ownership Dynamics, Structural Estimation, Stock Prices

JEL codes: G11, G12, G14, G32

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1. Introduction

Controlling shareholders have additional motives to hold shares in the company compared to minority shareholders, as they are able to extract private benefits such as social status, public prestige, or discretionary power to divert cash flows or to pay excessive compensation to blockholders or their relatives. Private benefits of control, then, can be an important driver in the controlling shareholders' choice about the size of the initial ownership share, and about their subsequent trading decisions. Since the controlling shareholders' trading decisions affect the formation of investors' beliefs and the amount of shares floating on the market, private benefits may have significant impact on stock prices.

In this paper, I present and estimate a dynamic model of optimal shareholding to quantify the private benefits of controlling shareholders and to measure the impact of private benefits of control on stock prices over time.

In the model, a large shareholder and a mass of marginal investors hold shares in a company. The marginal investors are uninformed about the fundamental value of the firm, and they trade on their heterogeneous expectations, which they revise over time using two pieces of information: the shocks to the fundamental value of the firm and the trading decision of the large shareholder, who has perfect information over the true value of the firm. However, the large shareholder also extracts private benefits from the stake, so that the information released by his trading is noisy.

The large shareholder trades off stock mispricing and private benefits against risk diversification and price impact of the trade, where the amount of private benefits extracted from the stake depends on the attainment of given thresholds of stake (for instance, 50% for the control of the firm). This assumption is in line with the institutional framework, according to which shareholders' rights and obligations arise as soon as shareholders get hold of a given percentage of the outstanding shares.

In the absence of private benefits, the large shareholder always trades on the mispricing of

the marginal investors. However, the mispricing reduces over time as the marginal investors learn from the large shareholder's trades. Thus, it becomes less profitable for the latter to exploit the mispricing. With additional private benefits, instead, the large shareholder may not trade at all. The reason is that the large shareholder sells a block of shares only when the share is largely overvalued, so that the gains offset both the loss in private benefits and shares depreciation, given the negative price impact of the trade. Specular reasoning applies to the purchase of an additional block of shares.

The price impact of private benefits is two-fold. First, private benefits affect the decision of the large shareholder in terms of size of the stake, and so the number of shares tradable on the market. Second, when the controlling shareholder extracts private benefits from the ownership share, his trading decisions are less affected by the fundamental value of the firm, and so they are less informative about the true value of the firm. Therefore, the presence of private benefits makes it more noisy for the rest of the investors to extract information from the large shareholder's trade.

To estimate the model, I use data on Italian public companies for which large shareholders are required to disclose their stakes every six months, thus allowing for data with higher frequency with respect to previous studies on the ownership dynamics (e.g., Donelli et al. (2013)).¹ The data show that large shareholders trade infrequently, despite substantial variation in the economic fundamentals of the firm, stock prices, and trading volumes. Moreover, when they trade, large shareholders buy or sell large blocks of shares, while they trade small stakes very rarely.

The main estimation results are the following. First, I estimate private benefits of control of around 2% of equity value. Moreover, I show that the distribution of private benefits is highly positively skewed: 50% of the controlling shareholders extract private benefits of less

¹In comparison, US disclosure rules allow to have information only on purchases of blocks above 5% of the outstanding shares (file 13D or 13G), and on the portfolio of big institutional investors with more than 100 millions of dollars of equity assets under management (file 13F).

than 1% of equity value, and the maximum rate is 20%.

Second, I show that private benefits of control generally have positive impact on stock prices. As for the private benefits, the price impact is very heterogenous across firms. For 15% of firms, in fact, private benefits have a negative impact greater than 1%, and for the same proportion of firms private benefits have a positive impact larger than 5%. Moreover, I find evidence of a synergistic effect. When the controlling shareholder is a corporation, the positive price impact of the large shareholder's stake is even larger compared to the case of individual controlling shareholders.

Third, I document that the presence of controlling shareholders has, overall, a substantial positive impact on stock prices. This positive price impact is larger during the European sovereign debt crisis of 2011-2012, so that controlling shareholders may be particularly beneficial to the rest of the investors during negative economic cycles.

Finally, I estimate the certainty equivalent payoff of the large shareholders' stake over time, which is unobservable when the large shareholder does not trade. The certainty equivalent payoff is the valuation of the share by an investor, and so the maximum price the investor would be willing to pay to buy the share.

My paper makes contributions to both theoretical and empirical studies on private benefits of control and controlling shareholders' ownership policy. To the best of my knowledge, this is the first paper to provide a measure of the impact of large shareholders on stock prices over time, only predicted in theory by the dynamic models of Collin-Dufresne and Fos (2015) and DeMarzo and Urosevic (2006), and estimated at the time of the block trade by Albuquerque and Schroth (2010).

Theoretical papers predict that the heterogenous valuation between large and small shareholders should always trigger trading by the large shareholder (Collin-Dufresne and Fos (2015), Hilli et al. (2013), DeMarzo and Urosevic (2006), and Gomes (2000)). The implication of persistent trading by the large shareholder is contradicted by empirical evidence showing that the frequency of trading by large shareholders is much less than expected from

the predictions of current models (e.g., Donelli et al. (2013)). The infrequent trading by the large shareholder, then, emerges as implication of my model with extraction of private benefits.

Further, I propose a new approach to quantify private benefits of control. Previous studies focus on the acquisition of the controlling stake to measure private benefits of control (e.g., Barclay and Holderness (1989), Nenova (2003), Nicodano and Sembenelli (2004), Dyck and Zingales (2004), and Albuquerque and Schroth (2010)). However, if private benefits may impact on the ownership policy of the controlling shareholder over time, then the ownership policy of the controlling shareholder may contain crucial information to estimate such benefits. The intuition is that private benefits make the controlling shareholder's trading less sensitive to changes in the economic conditions of the firm, since the incentive to hold the stake may be mostly due to the opportunity to enjoy benefits that are unrelated to the economic fundamentals of the firm. For this reason, I use data on the ownership dynamics of large shareholders to assess the magnitude of private benefits of control and their impact on stock prices.

My structural estimates of private benefits of control are in line with that of Albuquerque and Schroth (2010). Similarly to Albuquerque and Schroth (2010) and Nicodano and Sembenelli (2004), I show that the distribution of private benefits is highly positively skewed. My results show that 50% of the large shareholders (40% in Albuquerque and Schroth (2010)) extract private benefits less than 1% of equity value, and the maximum rate of private benefits is 20% (15%). Moreover, my estimates are generally in line with the evidence on private benefits of control in countries with large minority investors protection, such as anglo-saxon and north European countries. This results is consistent with Dyck and Zingales (2004), who show that private benefits of control in Italy dropped dramatically after the passage of a corporate governance reform (known as Draghi reform) that substantially increased minority shareholders protection.

By estimating a structural model, with endogenous asset pricing, I can quantify the price

impact of the private benefits of control, and of the presence of a controlling shareholder. Measuring the price impact requires a counterfactual analysis: what should be the stock price in the absence of private benefits? And, what should be the stock price in the absence of a controlling shareholder (i.e., completely dispersed ownership among atomistic investors)? I use the model pricing equations to quantify the difference between the actual stock price and the unobservable counterfactual stock price. Albuquerque and Schroth (2015) adopt the same methodology to quantify the price impact of illiquidity in the market for trading control blocks, and Lippi and Schivardi (2014) perform counterfactual analysis to show that private benefits of control have significant and negative impact on firm's profitability. Moreover, by using the optimality conditions of the controlling shareholder, I quantify the valuation of the stake by the controlling shareholder, thus addressing the question whether private benefits motivate controlling shareholders to hold large stakes even at the cost of excessive risk exposure.

Even though structural estimation relies on a specific theoretical model, I show that the estimated model performs well in replicating empirical facts on the dynamics of large shareholders' stakes that have not been explicitly targeted.

I review the relevant literature in the next section. In Section 1.3, I describe the theoretical framework and characterize the model equilibrium. Section 1.4 describes the estimation methodology, followed by the estimation results in Section 1.5. Section 1.6 concludes the chapter.

2. Literature Review

The theoretical framework of the paper builds on the literature that rationalises the ownership policy of a large shareholder (Collin-Dufresne and Fos (2015), Hilli et al. (2013), DeMarzo and Urosevic (2006), and Gomes (2000)). In Collin-Dufresne and Fos (2015), an activist shareholder accumulates shares and improves the firm's value as long as her stake is not fully revealed to the market. The large shareholder of DeMarzo and Urosevic (2006)

is prevented to trade to her optimal portfolio allocation from moral hazard, since marginal investors revise the stock price on the base of the large shareholder's stake. As a result, the large shareholder adjusts gradually the stake towards the optimal risk-sharing allocation. Gomes (2000) derives similar trading pattern for the owner-manager of a firm in the presence of asymmetric information, when the small investors do not perfectly observe the managerial ability of the large shareholder. Hilli et al. (2013) show that, even with separation between management and control, the large shareholder trades gradually to her optimal portfolio allocation if there is divergence of interests between manager and owner. The common point among these papers is that the friction triggers persistent trading by the large shareholder, while in the absence of the friction the large shareholder trades immediately to the optimal allocation.

My model assumes an exogenous ownership structure, with one large shareholder only and a mass of small dispersed investors, as in Collin-Dufresne and Fos (2015), Hilli et al. (2013), DeMarzo and Urošević (2006), and Gomes (2000). A few papers endogenise the ownership structure with different motivations. In the seminal work of Bolton and von Thadden (1998), inside investors trade-off the need of liquidity provided by potential external investors with the cost of being monitored by additional shareholders. In Zwiebel (1995), wealth constraints motivate shareholders to form a controlling coalition to undertake and finance investments, at the cost of sharing fixed private benefits of control. Several blockholders arise in the framework of Dhillon and Rossetto (2014) to mitigate the conflict of interests between the largest shareholder and the marginal investors. Yet, Dhillon and Rossetto (2014) show that one only big shareholder is optimal when such conflict is mild.

Many papers have documented ownership concentration across countries. Faccio and Lang (2002) show that families play a prominent role as controlling shareholder in Western European companies, and La Porta et al. (1999) report similar pattern across worldwide companies. Both papers find an inverse relationship between legal investors' protection and ownership concentration, consistently with the prediction of Dhillon and Rossetto (2014).

In particular, Laeven and Levine (2008) show that half of their sample firms have one large shareholder only and residual ownership widely held across dispersed shareholders.

Albeit quantitative measures of the controlling shareholder's impact on stock prices over time are still missing, several studies have highlighted the negative impact of the controlling shareholder on the risk taking of the company, in line with the theoretical results of Admati et al. (1994). Rossetto and Staglian (2018) find supporting evidence of this theoretical prediction only when the controlling shareholder is the only large shareholder of the company. John et al. (2008) argue that controlling shareholders undertake sub-optimally conservative investment decisions to preserve their private benefits of control, while Faccio et al. (2011) motivate the inverse relationship between control and risk with the large risk exposure of the controlling shareholder due to the large stake held in the firm.

Recently, Roger and Schatt (2016) have rationalised private benefits of control as compensation to controlling shareholders for excessive risk exposure in the controlled firm. Roger and Schatt (2016) show that the opportunity to enjoy private benefits motivate shareholders to hold the controlling stake, otherwise inefficiently large. Dyck and Zingales (2004) point out that control confers also costs in terms of low diversification of the controlling investor. However, Odegaard (2016) find little evidence of this argument. With a unique dataset of Norwegian equity-holders, Odegaard (2016) concludes that the magnitude of private benefits enjoyed by large investors does not compensate properly their diversification loss.

Empirical literature has followed two prominent approaches to estimate private benefits of control. The first approach is based on the price difference between shares with voting rights and shares with cash-flows rights. Zingales (1995) and Nenova (2003), among others, claim that the price premium of superior vote shares is justified by the perspective of future benefits of control. The price premium, however, can only be observed in firms that issue dual-class shares (i.e., voting rights and cash-flows rights). For this reason, Benos and Weisbach (2004) argue that this approach overstates private benefits of control, since firms with dual-class shares are likely to confer much larger amount of private control benefits.

The second approach is based on the difference between the price of the controlling stake and the market price of the shares at the day of the block negotiation. This difference is defined as block premium. Barclay and Holderness (1989), Nicodano and Sembenelli (2004), and Dyck and Zingales (2004) use the block premium as an empirical proxy to quantify the private benefits of control. First, they show that controlling shareholders are willing to pay more than the market price to buy the controlling stake of a company. Then, they argue that the premium is justified by the opportunity for the controlling shareholder to extract private benefits from the controlling stake, and try to disentangle from the premium the amount of (expected) private control benefits embedded in the price difference. Albuquerque and Schroth (2010) adopt a structural approach, by using the block pricing model of Burkart et al. (2000), to identify the private benefits of control transferred in the trade of the controlling block. Structural estimation is motivated by the great challenge to disentangle in the block premium the private benefits from the change in the share value due to the takeover of the incoming controlling shareholder.

A few papers have quantified private benefits of control in Italy. The first attempt of Zingales (1995), by using the voting premium approach, delivers very large estimates for Italian listed companies, on average around 80% of the equity value. The results of Nenova (2003) yield much lower numbers, claiming that Zingales (1995) computes private benefits for each single share in the block instead of taking the block as a whole. Nicodano and Sembenelli (2004), using a different approach which adjusts the methodology of Barclay and Holderness (1989) for the degree of dispersion of the ownership structure, provide evidence in line with Nenova (2003). Zingales (1995), Nenova (2003), and Nicodano and Sembenelli (2004), estimate private benefits of control before the Draghi reform of 1998, that increased the minority shareholders protection. Dyck and Zingales (2004) show that private benefits of control in Italy dropped on average from 47% to 6% after the Draghi reform.

3. The Model

In this section, I describe the model. First, I state the model assumptions, then I derive the optimality conditions for the marginal investors and the large shareholder. Moreover, I derive the equilibrium stock price and large shareholder's stake. Finally, I summarize the main results.

3.1. The Setup

The economy consists of investors with two investment opportunities: the shares of a company and a riskless asset. The following assumptions describe this economy.

- **Assumption 1. Investment Opportunities**

The firm is in unit supply and generates cumulative free cash flows described by the following diffusion

$$dD_t = \mu_t dt + \sigma_D dZ_t, \tag{1}$$

$$d\mu_t = \sigma dX_t, \tag{2}$$

where μ_t is a time-varying drift, σ and σ_D are constant, and dZ_t and dX_t are two independent standard Brownian motions. The firm pays out all cash flows as dividends. The riskless investment pays a continuously compounded rate of return r , with perfect elastic supply. Without loss of generality, I assume that all cash flows are paid to all shareholders each period, in forms of dividends, share repurchases or issuance. The assumption of normality on the cash flows implies that they can be negative. Moreover, they are independent across periods. Both independence and normality ensure tractability of the model.

I use the assumption of a time-varying drift to introduce asymmetric information between marginal investors and large shareholder, and the sequential learning of the

marginal investors in bayesian fashion, using the observed dividends flow as noisy signal on μ_t (see Assumption 3).

- **Assumption 2. Investors Population**

The economy is populated by a continuum of competitive investors, with measure M . All the investors are risk-averse, with standard CARA utility function, defined on the continuous flow of consumption. The agents have equal risk and intertemporal preferences. The investors live infinitely, and trade continuously the riskless asset and the company shares, with price P to be determined in equilibrium. Let $\alpha_{i,t}$ denote the number of shares owned at time t by the investor i , with i going from 1 to M . Each investor, then, chooses the amount of consumption and the number of shares to maximise

$$E_t \int_t^\infty e^{-R(s-t)} u(c_s) ds,$$

where $u(c) = -e^{-ac}$, a is the absolute risk-aversion coefficient, and R is the rate of intertemporal preferences, that is equal to r when $a = 0$. The wealth of each investor i is given by the riskless asset and the company shares, that is $W_i = B_i + \alpha_i P$. I assume that small shareholders cannot form coalitions and do not behave strategically.

There exists a large shareholder, who is risk-averse, with equal risk and intertemporal preferences as the marginal investors, lives infinitely, and has identical objective function as the marginal investors. However, the large shareholder differs from the rest of the investors in two directions. First, the large shareholder sets the optimal number of shares at discrete dates τ .² Moreover, the large shareholder extracts additional benefits

²This assumption improves the tractability of the model and follows DeMarzo and Urosevic (2006).

from investing in the firm, that accrue to the total wealth of the shareholder given by $W_L = B_L + \alpha_L P + \Phi(\alpha_L)$. The private benefits from shareholding, $\Phi(\alpha_L)$, generate continuously an instantaneous inflow of additional wealth for the large shareholder, denoted by $\phi(\alpha_L)$, such that $dW_L = d(B_L + \alpha_L P) + \phi(\alpha_L)$, according to a discrete step function:

$$\phi(\alpha_L) = b * \alpha_j,$$

if $\alpha_j \leq \alpha_L < \alpha_{j+1}$, where α_j and α_{j+1} are given thresholds of stakes (for instance, 0.2 and 0.3, respectively), with $j = \{0, 1, 2, \dots, J - 1\}$, $\alpha_0 = 0$, and $\alpha_J = 1$.

I make this assumption for consistency with the actual institutional framework, in which the shareholders' rights and obligations are triggered as soon as the shareholders come into possession of a given percentage of the outstanding shares. It follows that the shareholders acquire additional rights (or have the duty to comply with additional obligations) only if they reach the next higher threshold, and they lose rights (or are free from complying with a given obligation) as soon as they hold one stock less than a given percentage of shares. The private benefits take here the form of monetary incomes, or non-pecuniarity amenities that can be converted in additional wealth. b may take any real value, either positive or negative. Negative values imply that the large shareholder is bearing private costs from holding the stake in the company, in forms of outflow of money deducted from his total wealth.

- **Assumption 3. Heterogeneous Information and Beliefs Update**

The time-varying drift of the dividends pay out, denoted by μ_t , is unobservable to the marginal investors and observable only to the large shareholder. The marginal investors have heterogenous prior on μ_t , conditioning on the information set at time t , denoted

by $\mu_{i,t}$, and they have heterogenous prior variances, that are different conjectures on σ^2 , denoted by σ_i^2 . The marginal investors receive two types of signal to update their prior on μ_t , in a bayesian fashion.

At each time t , the investors observe the continuous cash flow dD_t , that is a noisy signal on μ_t . The signal is noisy as the investors are not able to disentangle between the pure shock on the dividend payout, given by σdZ_t , and the true dividend drift μ_t . Therefore, each investor updates continuously her prior on μ_t according to the conjecture on σ . In particular, the larger is the investor's prior variance σ_i^2 , the lower is the level of confidence of the investor about her prior, and the larger is the weight assigned to the noisy signal for revising the prior on μ_t . The investor's prior on μ_t , then, evolves according to the following equation

$$d\mu_{i,t} = k_i \eta_{i,t},$$

where, following standard bayesian filtering results,

$$k_i = \frac{\sigma_i^2}{\sigma_D^2 + \sigma_i^2}, \eta_{i,t} = dD_t - E_{i,t}[dD_t],$$

and $E_{i,t}[dD_t] = \mu_{i,t}$.

Therefore, the heterogeneity across investors is fully described by the distribution of the coefficient k_i , that is the weight assigned to the signal for updating the beliefs on the expected dividend payout.³

³The reader can refer to a set of investors with different level of confidence or information on the future cash flows generated by a company. An investor with superior level of information, or high degree of confidence, has a low value of k , and relies little on the signals provided by the actual dividend payments. Instead, a poorly informed investor is easily conditioned by the fresher information provided by the new dividend payment, and has a high value of k .

At the discrete dates τ , the investors observe the ownership share of the large shareholder $\alpha_{L,\tau}$, that is a noisy signal on μ_t , since investors know that the large shareholder has additional motives to invest in the firm, given by the private benefits, that are unobservable to the marginal investors. The update at τ of each marginal investor about μ_t is then described by the following equation:

$$\mu_{i,\tau} = \mu_{i,t < \tau} + g_i(\tau^-)(\alpha_{L,\tau} - \alpha_{L,\tau}(\mu_{i,t < \tau})),$$

where $\mu_{i,t < \tau}$ is the prior of the investor i before observing the choice of the large shareholder, $\alpha_{L,\tau}(\mu_{i,t < \tau})$ is the belief of the investor i about the optimal choice of the large shareholder, and $g_i(\tau^-)$ is the weight assigned to the observation of the large shareholder's choice for updating the prior on μ_t . Therefore, each investor i revises upwards (downwards) the conjecture on the true dividends drift of the firm when the large shareholder sets an ownership share above (below) the expected choice, that is interpreted as a positive (negative) signal on the true state of the firm. Following again standard bayesian filtering results, the weight $g_i(\tau^-)$ is equal to

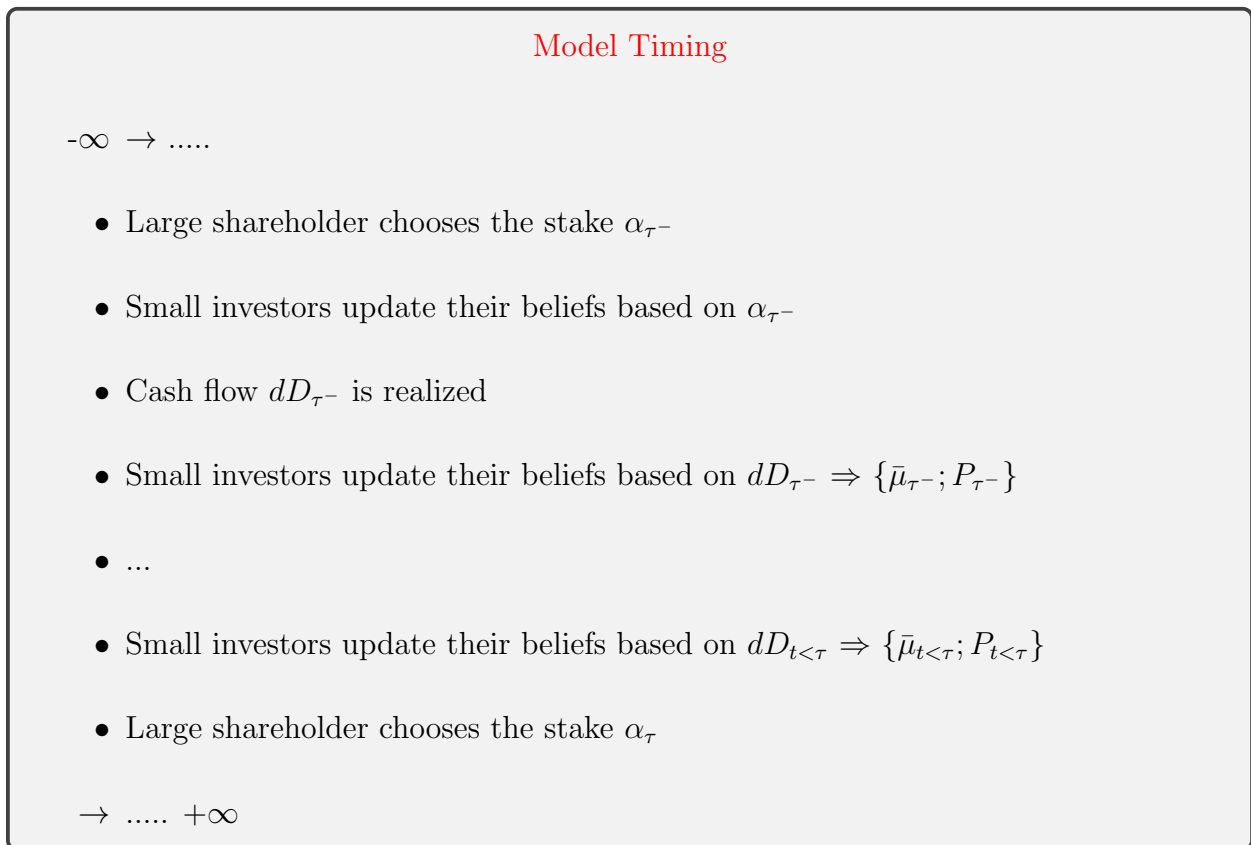
$$g_i(\tau^-) = \frac{\alpha_{L,\tau^-}^\mu \sigma_i^2}{(\alpha_{L,\tau^-}^\mu)^2 \sigma_i^2 + \sigma_\epsilon^2},$$

where $\alpha_{L,\tau}^\mu = \frac{\partial \alpha_{L,\tau}}{\partial \mu_t}$ is the derivative of the optimal choice of the large shareholder with respect to the true dividends drift, and σ_ϵ^2 is the variance of the observation error. So, σ_ϵ^2 is a measure of the noise contained in the information on μ_t released by the large shareholder with his optimal choice.

In case of no private benefits, the marginal investors know that the large shareholder sets the demand for shares only on the base of the superior information on the dividends drift. Therefore, the observation α_{L,τ^-} is clean, that is $\sigma_\epsilon^2 = 0$, so $g_i(\tau^-) = \frac{1}{\alpha_{L,\tau^-}^\mu}$, where α_{L,τ^-}^μ is observable, albeit with one period lag due to asymmetric information.

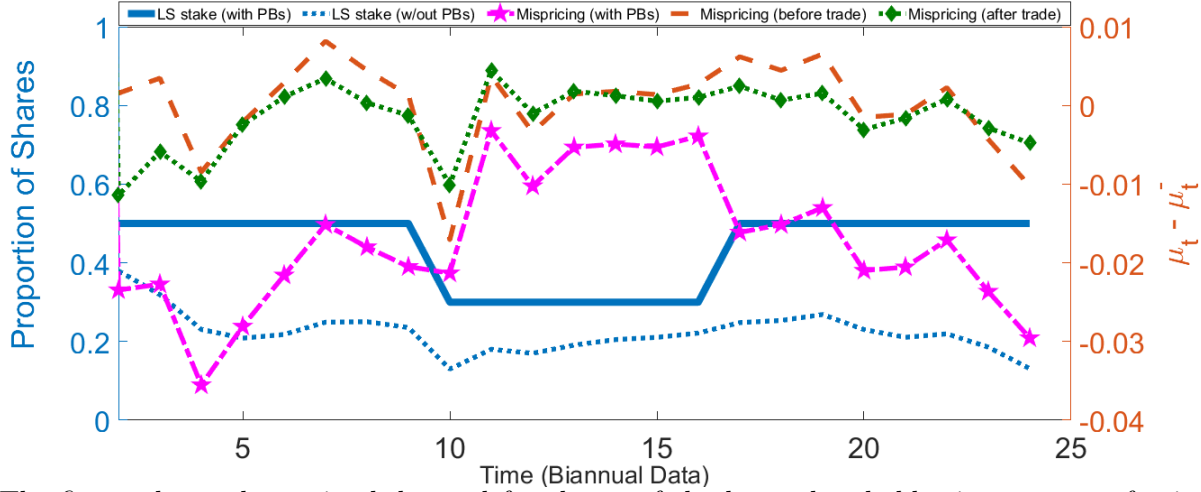
With private benefits, the observation α_{L,τ^-} is not longer clean, that is $\sigma_\epsilon^2 > 0$.

Given assumptions 1, 2, and 3, the following sections characterize the model solution. First, I summarize the main results in figure 1, with a graphical simulation. Details on the simulation study are provided in the Appendix. Then, I describe analytically the investors' optimality and the equilibrium stock price, the large shareholder's optimality, finally deriving the model equilibrium. I leave the proofs for the Appendix to save in space and notation. Next, I depict the timing of the model.



In Figure 1, the dotted line shows that in the absence of private benefits the large shareholder always trades, to exploit the mispricing of the marginal investors (dashed line). The mispricing is given by the difference between the true dividends drift and the average belief on the dividends drift by the marginal investors. However, the marginal investors learn

Figure 1. Ownership Dynamics, Private Benefits and Mispricing



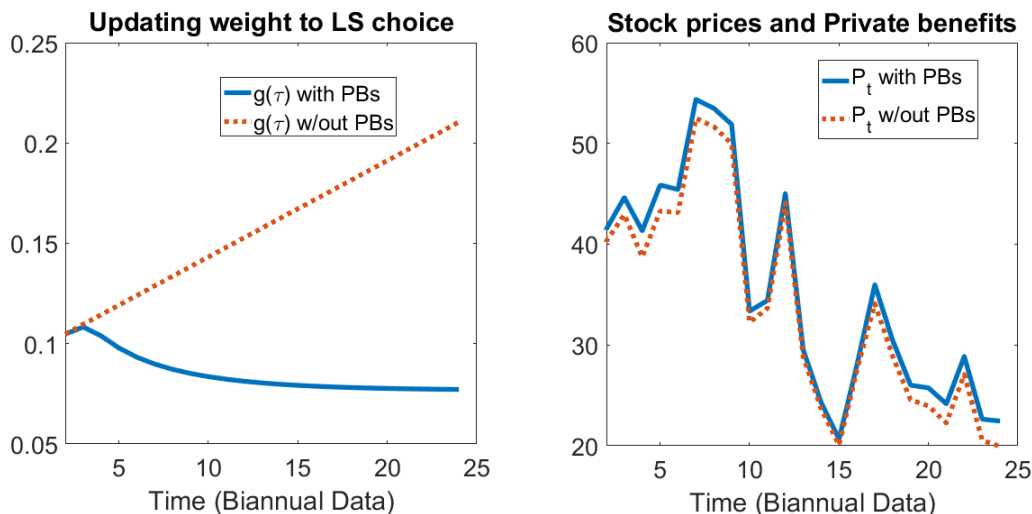
The figure shows the optimal demand for shares of the large shareholder in presence of private benefits (blue line), and in absence of private benefits (dotted line), against the difference between the true dividends drift (μ_t) and the average belief on the dividends drift ($\bar{\mu}_t$) by the marginal investors before the disclosure of the large shareholder's stake (dashed line), after the disclosure of the large shareholder's stake in absence of private benefits (diamond line), and after the disclosure of the large shareholder's stake in presence of private benefits (stars line). The parameters used for the numerical example are the same as for the simulation study described in the Appendix.

from the large shareholder's trade revise their belief after observing the large shareholder's trade (diamond line). As a result, the large shareholder trades gradually to the optimal risk-sharing solution.

With private benefits, the large shareholder trades only at two points in time (blue line). The large shareholder sells (buys) a block of shares when the overvaluation (undervaluation) by the marginal investors makes the sale (purchase) convenient to the large shareholder: the trading gains (costs) offset both the loss (gain) in private benefits and the share depreciation (appreciation), due to the negative (positive) price impact of his trade.

Yet, in the presence of private benefits, the signal released by the large shareholder with his demand for shares is noisy. The noise in the information released by the large shareholder's demand generates a distortion in the update of the marginal investors' belief (stars line), and the marginal investors consider less reliable this information in the update

Figure 2. The impact of LS ownership policy



The left panel shows the average updating weight assigned to the observation of the large shareholder’s stake by the marginal investors to update their belief on the dividends drift, with (blue line) and without (red dotted line) private benefits. The right panel shows the equilibrium stock price at the disclosure dates τ with (blue line) and without (red dotted line) private benefits. The parameters used for the numerical example are the same as for the simulation study described in the Appendix.

of their belief compared to the case of no private benefits. The left panel of Figure 2 shows that the weight assigned by the marginal investors to the information released by the large shareholder’s demand ($g(\tau)$) in the presence of private benefits is gradually lower compared to the case of no private benefits.

3.2. Equilibrium Share Price and Investor’s Optimality

I start characterizing the model solution describing the marginal investors’ optimality conditions.

At each time t , the optimal choice in terms of number of shares of the investor i is given by:

$$\alpha_{i,t} = \frac{\mu_{i,t} - \bar{\mu}_t + \rho}{ar\sigma^2}, \quad (3)$$

where $\bar{\mu}_t$ is the average expected dividend by the marginal investors, and the risk premium ρ , determined by market clearing condition, that is $\int_i \alpha_{i,t} di = 1 - \alpha_{L,t}$ for each t , is

$$\rho_t = (1 - \alpha_{L,t})a^I r \sigma^2,$$

where $\alpha_{L,t}$ is the stake held by the large shareholder at time t , and a^I is the aggregate risk aversion coefficient:

$$\frac{1}{a^I} = \int_i \frac{1}{a^i} di.$$

Proof. Appendix 1.7.1

Equation (3) has a natural interpretation. While the demand for shares decreases with the dividends process variance and the risk aversion coefficient, the demand increases with the difference between the individual and the average belief about the expected dividend payout of the firm. Equation (3) reminds the familiar optimal risky asset allocation for a mean-variance investor, and it is equivalent to the optimal solution of the small price-taker investor of DeMarzo and Urosevic (2006). However, in DeMarzo and Urosevic (2006) the risk premium only depends on the large shareholder's trading, as the expected dividend payout is observable.

Next, I derive the equilibrium share price. At each time t , the stock price is the following:

$$P_t = \int_t^\infty e^{-r(s-t)} (\bar{\mu}_s - \rho_s) ds. \quad (4)$$

Proof. Appendix 1.7.2

Hence, the share price is the present value of the expected dividends by the marginal investors, minus the risk premium component. The former evolves continuously according to the following equation:

$$d\bar{\mu}_t = \bar{k}_t(dD_t - \bar{\mu}_t). \quad (5)$$

Proof. Appendix 1.7.3

Equation (5) shows that the average belief increases (decreases) when the actual dividend payout is greater (lower) than the expected dividend payout by the marginal investors, and the rate of growth is proportional to the average reaction of the marginal investors to the new signal. Then, share prices fluctuate also independently from the economic fundamentals of the company, due to over or under reaction of the investors to news and shocks. Chan (2003) shows that investors react slowly to valid information, while they overreact to price shocks, causing huge trading volume and price volatility.

3.3. Large Shareholder's Optimality

Given the average prior on the expected dividend payout by the marginal investors $\bar{\mu}_t$, reflected in the share price $P_t = P(\bar{\mu}_t)$, the large shareholder chooses at discrete dates τ the optimal number of shares, $\alpha_{L,\tau}$, to maximize the certainty equivalent payoff

$$V(\alpha_{L,\tau}) - (\alpha_{L,\tau} - \alpha_{L,\tau^-})P_\tau,$$

where

$$V(\alpha_{L,\tau}) = \int_\tau^\infty e^{-r(s-\tau)} v(\alpha_{L,s}) ds,$$

and

$$v(\alpha_{L,t}) = \alpha_{L,t}\mu_t - \frac{1}{2}\alpha_{L,t}^2 a^L \sigma_D^2 r + \phi(b, \alpha_{L,t}).$$

$v(\alpha_{L,t})$ is the net benefits flow to the large shareholder at each time t , given by the risk-adjusted instantaneous dividend process accrued to the large shareholder, plus the additional inflow of instantaneous private benefits generated by the stake.

So, the certainty equivalent payoff is given by the present value of the net benefits flow

less (plus) the trading costs (gains), where α_{L,t^-} stands for the number of shares at the previous point in time. By taking the first-order derivative with respect to $\alpha_{L,t}$, I obtain the large shareholder's optimality condition:

$$V' = P_\tau + (\alpha_{L,\tau} - \alpha_{L,\tau^-})P', \quad (6)$$

where the prime index stands for the derivative with respect to the control variable. Equation (6) is the usual equilibrium condition that equates benefits and costs, where the marginal benefits are given by the risk-adjusted cumulative expected dividends plus the private benefits generated by an additional share, and the marginal costs are given by the share price plus the implicit cost of trading, due to the price impact of the large shareholder's stake.⁴

3.4. Equilibrium

I now characterize the equilibrium stock price and the optimal demand for shares of the large shareholder, taking into account his impact on the stock price. The equilibrium risk premium is derived by the market clearing condition in the static context (sum of investors' demand for shares equal to total number of shares minus the large shareholder's stake), while the average expected dividend payout is derived by the market clearing condition in the dynamic context (sum of shares purchased equal to sum of shares sold).

Proposition 1. At each discrete date τ , the equilibrium share price is given by

$$P_\tau = \int_\tau^\infty e^{-r(s-\tau)}(\bar{\mu}_\tau - \rho_\tau)ds,$$

where the equilibrium risk premium is $\rho_\tau = (1 - \alpha_{L,\tau}) * a^I \sigma_D^2 r$, and

⁴Given the step function characterizing the private benefits of the large shareholder, the objective function displays local maxima at the thresholds, thus implying that the large shareholder in equilibrium holds a stake equal to either one of the thresholds.

$$\bar{\mu}_\tau = \bar{\mu}_{t<\tau} + \bar{g}(\tau^-)(\alpha_{L,\tau} - \alpha_{L,\tau}(\bar{\mu}_{t<\tau})).$$

Proof. Appendix 1.7.4

$\bar{\mu}_\tau$ is the average posterior belief on the dividend payout of the firm by the marginal investors, after the large shareholder's choice of ownership share, where $\bar{g}(\tau^-)$ is the average reaction by the marginal investors to the large shareholder's choice of ownership share, and $\alpha_{L,\tau}(\bar{\mu}_{t<\tau})$ is the average prior by the marginal investors about the optimal choice of the large shareholder:

$$\alpha_{L,\tau}(\bar{\mu}_{t<\tau}) = \frac{(1 + \alpha_{L,\tau^-})a^I \sigma_D^2 r}{2a^I \sigma_D^2 r + a^L \sigma_D^2 r},$$

where $\bar{\mu}_{t<\tau} = \bar{\mu}(dDt)$ is the average expected dividend payout by the marginal investors before the large shareholder's choice of ownership share, which is function of the continuous signals given by the company's dividends.

Proposition 2. Taking into account his price impact, the large shareholder's optimal demand for shares is the following

$$\alpha_{L,\tau} = \frac{\mu_\tau - \bar{\mu}_{t<\tau} + \phi'(b, \alpha_{L,\tau})}{2a^I \sigma_D^2 r + a^L \sigma_D^2 r + \bar{g}(\tau^-)} + \alpha_{L,\tau}(\bar{\mu}_{t<\tau}).$$

Proof. Appendix 1.7.5

The large shareholder's optimal demand for shares is the sum of three components: the speculation on the investors' mispricing, the expected optimal choice of shares conjectured by the marginal investors, and the gain in the private benefits generated by an additional share, which is positive only when the additional share allows to reach a higher threshold. The next section provides closed-form solution in two benchmark cases, and provides further details on the solution in the general case.

3.5. Main Results

- **Perfect Information and No Private Benefits**

In case of perfect information and no private benefits, the marginal investors set the weight to assign to the observation of the large shareholder's choice $\alpha_{L,\tau}$ at time τ , then $\bar{g}(\tau)$ is simply equal to $1/(2a^I\sigma_D^2r + a^L\sigma_D^2r)$, so that $\bar{\mu}_t = \mu_t$, since

$$\alpha_{L,\tau} - \alpha_{L,\tau}(\bar{\mu}_{t<\tau}) = \frac{\mu_\tau - \bar{\mu}_{t<\tau}}{2a^I\sigma_D^2r + a^L\sigma_D^2r}.$$

When all the investors have the same set of information, at each point in time, then the equilibrium stock price is simply given by the present value of the expected dividends payout of the firm, given the true dividends drift μ_t , minus the equilibrium risk premium $\rho = (1 - \alpha_{L,t})a^I r \sigma^2$, where

$$\alpha_{L,t} = \frac{a^I}{a^L + a^I},$$

that is the large shareholder *immediately* trades to the risk-sharing allocation. This result is equivalent to DeMarzo and Urosevic (2006) in the absence of moral hazard.

- **Asymmetric Information and No Private Benefits**

With asymmetric information and no private benefits, the large shareholder's demand for shares is informative on the true drift of the dividends process. However, the marginal investors set the weight to assign to the observation of the large shareholder's choice $\alpha_{L,\tau}$ at time τ^- , so that

$$\bar{g}(\tau) = \frac{1}{\alpha_{L,\tau}^\mu} = \frac{1}{2a^I\sigma_D^2r + a^L\sigma_D^2r + \bar{g}(\tau^-)}.$$

In turn, at τ , the large shareholder knows and anticipates the response of the investors, thus taking into account the impact of his demand for shares. Given his superior information, the large shareholder can exploit the mispricing of the marginal investors, thus trading on the difference between his stock valuation and the average belief by the marginal investors. Therefore, the large shareholder has always the temptation to trade.

However, as the marginal investors learn from the large shareholder's trade, the stock price increases when the large shareholder buys, and falls when the large shareholder sells. This mechanism makes convex (concave) the trading costs (profits) on the purchase (sale) of the shares to the large shareholder, and prevents the large shareholder from trading to his first-best solution, that is the trading policy in the absence of the investors' learning.

As a result, the large shareholder trades *gradually*, yet not monotonically, at the discrete dates τ , towards the optimal risk-sharing allocation, while the marginal investors update their belief on the expected dividends payout. Indeed, the magnitude of the large shareholder's trade, and so the speed of the adjustment towards the optimal risk-sharing solution, is inversely proportional to the average reaction of the marginal investors ($g(\tau^-)$) to his trade. This result is equivalent to DeMarzo and Urošević (2006) in the presence of moral hazard.

- **Asymmetric Information and Private Benefits**

Finally, with private benefits, the information released by the large shareholder with his optimal demand for shares is noisy with respect to the true drift of the dividends process, then

$$\bar{g}(\tau) = \frac{\alpha_{L,\tau}^\mu \sigma_i^2}{(\alpha_{L,\tau}^\mu)^2 \sigma_i^2 + \sigma_\epsilon^2},$$

where the actual $\alpha_{L,\tau}^\mu$ is not observable, and therefore proxied by $1/(2a^I\sigma_D^2r + a^L\sigma_D^2r)$.

While in the absence of private benefits the large shareholder always trades at the discrete dates τ , in the presence of private benefits the large shareholder may not trade at all. The no-trade occurs when the large shareholder's stake is at a given threshold ($\alpha_{L,\tau} = \alpha_j$). The purchase of a share would not produce any additional private benefit while making even more undiversified and suboptimal the investment portfolio of the large shareholder. On the other side, selling a share produces a loss in private benefits ($\phi(\alpha_{L,\tau} < \alpha_j) = b * \alpha_{j-1}$) which may not be compensated by the gain in risk diversification.

The large shareholder, instead, sells a block of shares when the difference in the valuation, against the marginal investors, is negative (the share is overvalued) and large enough to make convenient the trade, even if this happens at the cost of losing a given amount of private benefits, and the shares depreciate because of the sale, given the price impact of his sale. On the other hand, the large shareholder is willing to buy an additional block of shares, in order to reach the higher threshold that generates additional private benefits, only when the difference in the valuation becomes positive (the share is undervalued) and large enough, taking into account the implicit cost of trading due to the share appreciation.

3.6. Price Impact of Private Benefits

Private benefits affect the equilibrium stock price in two directions. As shown above, the equilibrium stock price depends on both the average expected dividend by the marginal investors following the large shareholder's demand for shares, and the equilibrium risk premium ρ derived by the market clearing condition

$$P_\tau = P(\bar{\mu}(\alpha_{L,\tau}), \rho(\alpha_{L,\tau})).$$

Then, it is straightforward to note that private benefits increase the equilibrium stock price by reducing the equilibrium risk premium. In fact, private benefits increase the optimal demand for shares of the large shareholder, thus reducing the equilibrium risk premium sought by the marginal shareholders to invest in the firm. In practice, a larger demand for shares of the large shareholder reduces the number of shares tradable on the market, under the assumption of fixed shares supply, thus raising the stock price.

Moreover, private benefits affect the update, at τ , of the belief of the marginal investors on the true expected dividends payout, following the observation of the large shareholder's decision in terms of ownership share. First, since private benefits drive the optimal demand for shares of the large shareholder, then private benefits impact on the distance between the actual and the expected choice, by the marginal investors, of the large shareholder's ownership share:

$$(\alpha_{L,\tau} - \alpha_{L,\tau}(\bar{\mu}_{t < \tau})).$$

Further, private benefits affect the weight assigned by the marginal investors to the observation of the large shareholder's choice. Since this choice is driven by additional motives with respect to the dividend payout of the company, then the choice is a noisy signal on the dividend payout of the company, and so the marginal investors assign a lower weight to that observation in the update of their belief.

4. Structural Estimation

In this section, I report the actual data used for the estimation and I state the estimation problem, by describing the quantities to estimate, the parameters that are calibrated, and the observable variables involved in the estimation. Then, I describe the identification process, which links the observable variables to the unobservable quantities of the model.

4.1. Data

The universe of firms consists of the public companies listed in the Italian Stock Exchange, in which they are classified by market capitalization. I consider all the non-financial firms listed in the Large, Medium, and Small Capitalization indexes. The source of data for the ownership share of the large shareholders is Thomson Reuters Eikon, which combines public and private information on the ownership structure of public companies. However, I double-check manually the data by using the website of the Consob, the Italian security exchange commission, that releases information on the ownership structure of the public companies every six months, on the base of the company disclosure. So, I use biannual data on the largest shareholder's stake between March 2004 and September 2016 (24 observations). Thomson Reuters Datastream, on the other hand, provides also data on stock prices and earnings per share.

My final sample is obtained by selecting only the firms reporting the same largest shareholder for at least 75% of the observations. This filter allows to identify correctly the controlling shareholder of the firm. Further, I delete the firms with missing data over the time series. The final sample consists of 78 firms, 936 firm-year data on the earnings-per-share, 1,872 firm-biannual data on shareholdings and stock prices, and 280,800 firm-day observations on daily stock prices.

The large shareholder's average and median stake is around 50%, and it is quite stable over time (Figure 3, left panel). In fact, the trading activity of the large shareholder is very low. I observe a trade in only 18% of the total observations, where I refer to trade as a non-zero difference between two consecutive stake observations, and the average number of trades across firms is slightly above 3 (out of 24 observations for each firm). The right panel of Figure 3 shows the distribution of the number of trades across the largest shareholders in the sample.

Moreover, when they do, large shareholders usually trade big blocks of shares: the mean (median) trade is 4.27% (1.17%) of the outstanding shares. Finally, they rarely trade small

Table 1. Descriptive Statistics

	Variable	Mean	Median	Std	10th pct	90th pct
Company Data	Total Assets	1.24	0.15	3.42	0.01	2.79
	Debt-To-Equity	1.72	0.87	14.77	0.11	2.45
	Earning-Per-Share	0.30	0.14	1.02	-0.25	1.17
	FCF-Per-Share	0.94	0.48	1.49	-0.01	2.39
Stock Data	Daily Returns (%)	1.48	-0.38	20.92	-19.32	22.83
	Daily Turnover	0.38	0.23	0.60	0.02	5.26
LS Data	Stake (%)	48.63	53.29	17.53	18.88	66.96
	Trade (%)	4.27	0.41	1.17	0.00	4.35
	N of Trades	3.33	3	2.34	1	6

The table reports the descriptive statistics at company, stock, and largest shareholder levels. The statistics are the mean, the median, the standard deviation, the 10th and the 90th percentiles, computed over the 78 final sample, between March 2004 and September 2016. Company data are on annual basis, and are the total value of assets (in millions of euro), the debt-to-equity ratio, the earnings-per-share, and the free cash flow-per-share. Stock data are on daily basis, and are the returns and the number of shares traded divided by the number of outstanding shares. Largest shareholder data are on biannual basis, and are the percentage of shares held by the largest shareholder, the variation over time of the largest shareholder's stake, and the number of times the stake of the largest shareholder changes.

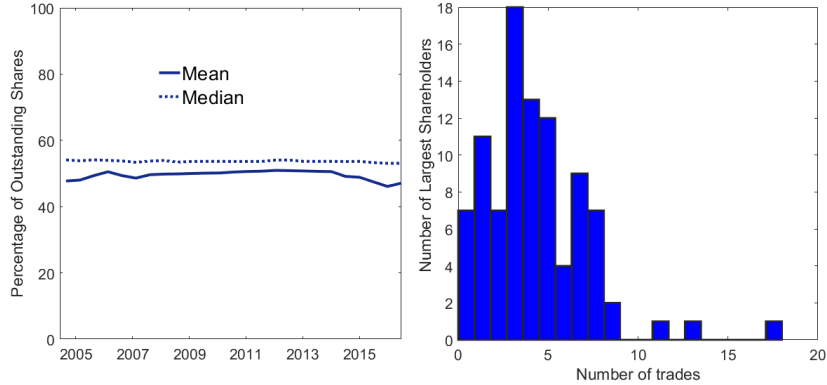
blocks of shares. I observe a trade that involves less than 1% of the outstanding shares in only 7.79% of total observations.

In summary, large shareholders show an ownership dynamics quite stable over time, and they trade very infrequently, usually buying or selling large blocks of shares. By contrast, the sample firms are characterised by a large trading volume over time, and stock prices fluctuate significantly, thus showing a substantial trading activity of the mass of shareholders and investors operating in the market. On average, 0.38% of the outstanding shares are traded every day.

4.2. Parameters Calibration

The steps of the private benefits function (α_j), and the aggregate risk aversion coefficient (a^I), are calibrated to the Italian data, for consistency with the actual dataset used for the estimation. I set a^I equal to 2, following Guiso et al. (2018) who measure the aggregate risk

Figure 3. Largest Shareholders: Stake and Trading



The left panel shows the mean (blue line) and the median (dotted line) ownership share, as percentage of outstanding shares, across the largest shareholders of the final 78 sample firms, between March 2004 and September 2016, at biannual frequency. The right panel shows the distribution of the number of trades of the largest shareholders of the final 78 sample firms, where a trade is defined as a non-zero difference between two consecutive stake observations.

aversion on a large set of clients of an Italian bank. The private benefits thresholds follow the Italian law on the ownership structure of public listed companies. The thresholds are listed in table II.

The rights (and the obligations) linked to each stake level are triggered *as soon as* the shareholder reaches a given threshold. This rule motivates the assumption that the stake generates a given amount of private benefits for a given threshold only if the stake is greater or equal than that threshold, while holding a stake even one share lower than a threshold generates private benefits according to the lower threshold (if $\alpha_L = 29.99\%$, then $\phi(\alpha_L) = b * (10\%)$).

The following figure shows the empirical distribution of the no-trade thresholds. I define no-trade threshold the percentage of shares at which the largest shareholder of the company does not trade, that is the largest shareholder holds that ownership share for at least two consecutive observations. Note that the largest number of no-trade thresholds are observed around 30%, and between 50% and 70%.

Table 2. Ownership Share Thresholds

Ownership Share	Right/Commitment
3%	Obligation to stake disclosure
10%	Right to call shareholders meeting
30%	Obligation to launch takeover
50%	Company control
66%	Right to call extraordinary meeting
100%	Full ownership

The table describes the ownership share thresholds that a shareholder has to attain in order to acquire a given right, or that triggers a given commitment, according to the Italian commercial law. The thresholds are expressed as percentage of the outstanding shares.

4.3. Structural Parameters and Latent Variables

I estimate the model firm-by-firm. For each firm, I estimate the following set of parameters:

$$\theta = \{a^L, \sigma_D, \sigma, b, \bar{g}(0), \sigma_\epsilon^2\},$$

that is the large shareholder's absolute risk aversion coefficient, the volatility of shocks to dividends, the volatility of shocks to dividends drift, the parameter that quantifies the private benefits extracted by the large shareholder, for a given discrete step function, the initial weight assigned to the large shareholder's demand for shares by the marginal investors to update their belief on μ_t , and the noise contained in the information released by the large shareholder's demand.

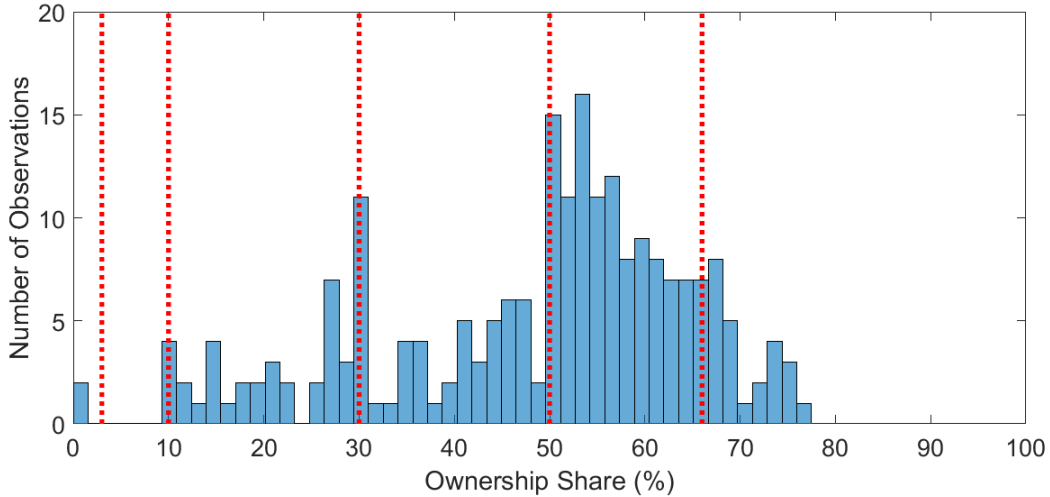
Moreover, for each firm, I infer the dynamics of the following set of latent variables:

$$X_t = \{\mu_t, \bar{\mu}_t\},$$

that includes the true dividends drift and the average expected dividends drift by the marginal investors.

I estimate the model by using stock prices, ownership share of large shareholders, and

Figure 4. Empirical No-Trade Thresholds



The figure shows the distribution of the no-trade ownership shares observed on the 78 final sample firms. The no-trade share is defined as the ownership share observed at least for two consecutive observations for a given largest shareholder, that is the ownership share at which the largest shareholder does not trade at least across two periods. The red vertical dotted lines are the stake thresholds described in Table II.

earnings-per-share.

4.4. First Step

In the first step, I estimate $\{\sigma_D, \sigma\}$ by using daily stock prices, that proxy the continuous evolution of the equilibrium share price in the model. To estimate, I discretize the equations (5) and (2), respectively, take the conditional expectation at time t , and derive the following set of diffusion equations that link the two latent variables:

$$E_t[\bar{\mu}_{t+1}] = (1 - \bar{k}_t)\bar{\mu}_t + \bar{k}_t\mu_t, \quad (7)$$

where k_t measures the average reaction to the new observation of the dividends payout by the marginal investors, and

$$E_t[\mu_{t+1}] = \mu_t, \quad (8)$$

as $E_t[dD_t] = \mu_t$, that is the true drift of the dividends process, and $E_t[d\mu_t] = 0$. The conditional covariance matrix of the two latent variables is diagonal⁵. The diagonal matrix depends on both σ_D and σ :

$$\Sigma_{t+1|t}(X_t) = \text{Diag}(\sigma^2, \bar{k}_t \sigma_D^2).$$

On the other hand, the stock price is linked to the two state variables according to the equation (4), that can be written as follows

$$P_t = \frac{1}{r} [(\sigma_D^2 a^I (1 - \alpha_{L,t})) + \bar{\mu}_t]. \quad (9)$$

Therefore, given a prior on μ_t and $\bar{\mu}_t$, I can compute a predicted stock price at each time t , by using the above equation, then obtaining a prediction error

$$e_t = \tilde{P}_t - \hat{P}_t,$$

where \tilde{P}_t is the actual stock price, and \hat{P}_t is the predicted stock price. The errors are function of the structural parameters and the latent variables, i.e. $e_t = e(a^I, \sigma_D^2, X_t)$, and the covariance matrix of the prediction errors depends on the derivative of the stock price with respect to the state variables and the conditional covariance matrix of the state variables:

⁵In the next formula, k_t is allowed to vary over time to proxy the evolution of the average reaction of the marginal investors to the new signal provided by the firm's payout:

$$k_t = \frac{\nu_t}{\nu_t + \sigma_D^2},$$

where $\nu_t = w_t + \sigma$, and at each time step w_t is updated by using $(1 - k_{t-1})\nu_{t-1}$, and initializing the recursion with a large value of ν_0 . This procedure allows to proxy the prior update on the dividends drift across the marginal investors.

$$\Sigma(e) = f \left(\frac{\partial P_t}{\partial X_t}, \Sigma_{t+1|t}(X_t) \right),$$

where

$$\frac{\partial P_t}{\partial X_t} = \left[\frac{1}{r} \bar{k}_t; \frac{1}{r} \right].$$

So, I construct a likelihood function on the prediction errors, under the assumption of normality, that I maximize with respect to σ :

$$\hat{\sigma} = \operatorname{argmax}_{\sigma} \ln \ell(e_t; \sigma) = -\frac{1}{2} \sum_{t=0}^T \ln |\Sigma(e)| - \frac{1}{2} \sum_{t=0}^T e_t' \Sigma(e)^{-1} e_t,$$

under the restriction that

$$\hat{\sigma}_D = \operatorname{argmin}_{\sigma_D} \left[\sigma_D - \sqrt{\frac{\operatorname{var}(\delta(D_t)) - \hat{\sigma}^2}{2}} \right]^2,$$

where $\delta(D_t)$ stands for the innovations in the earnings-per-share, and the above condition is derived from equation (1), noting that

$$\operatorname{var}(\delta(dD_t)) = \operatorname{var}(\delta(d\mu_t)) + 2\operatorname{var}(dD_t - \mu_t) = \sigma^2 + 2\sigma_D^2.$$

The maximization of the likelihood function, combined with the condition on the variance of the innovations in the earning-per-share, allows to simultaneously estimate σ_D and σ , and to infer the dynamics of the two state variables. The second result is achieved by iterating the updating and the predicting equations of the linear Kalman filter.⁶ In particular, for each time t , the prior estimate on X_t is updated on the base of the prediction error, thus obtaining a posterior estimate of X_t in a bayesian fashion, that is used as prior estimate for

⁶Details on the Kalman filter implementation, and details on the identification of k_t are provided in the appendix.

the next point in time.

4.5. Second Step

In the second step, I estimate $\{a_L, \bar{g}(0), \sigma_\epsilon^2\}$ by using biannual stock prices and contemporaneous ownership share of large shareholders. By using the model equilibrium conditions, I arrive at one equation that describes the stock price at the discrete dates τ , when the large shareholder discloses his ownership share, as function only of the exogenous variables and the ownership share of the large shareholder, by eliminating all the remaining endogenous quantities determined in the model.

Given $\bar{\mu}_{t<\tau}$ that I have estimated in the previous step with daily stock prices, the evolution of the biannual stock prices is endogenously determined using $\alpha_{L,\tau}$, $\bar{g}(\tau)$, and $\alpha_{L,\tau}(\bar{\mu}_{t<\tau})$, where $\bar{g}(\tau)$ is function of the exogenous parameters and $\bar{g}(\tau^-)$, and $\alpha_{L,\tau}(\bar{\mu}_{t<\tau})$ is function of the exogenous parameters and α_{L,τ^-} .

Hence, using again the equation (9), and similar approach to the previous step, I can compute a predicted stock price at each date τ , where I use $\bar{\mu}_\tau$, as determined in the model, to form my prediction on the stock price, thus obtaining again a prediction error at each date τ . Once more, the errors are function of the structural parameters and the latent variable $\bar{\mu}_\tau$. The conditional variance of the state variable is now simply $\bar{k}_t \sigma_D^2$, while the variance of the prediction errors depends again on the derivative of the stock price with respect to the state variable, now simply given by $1/r$, and the conditional variance of the state variable. Then, I construct a likelihood function on the prediction errors, under the assumption of normality, that I maximize with respect to $\{a_L, \bar{g}(0), \sigma_\epsilon^2\}$.

4.6. Identifying Private Benefits

Now, I describe the identification process of the private benefits parameter b . In particular, I derive upper and lower bounds for b , for each firm-large shareholder. First, let $J_\tau^L = J(\alpha_{L,\tau}, b)$ denote the maximal certainty equivalent payoff for the large shareholder at each time τ , given the optimal demand for shares $\alpha_{L,\tau}$

$$J_\tau^L = \int_\tau^\infty e^{-r(s-\tau)} v_s^*(\alpha_{L,\tau}) ds - (\alpha_{L,\tau} - \alpha_{L,\tau^-}) P(\alpha_{L,\tau}),$$

where $v^*(\alpha_{L,\tau})$ is the maximal net benefits flow to the large shareholder, given the optimal demand for shares $\alpha_{L,\tau}$.

J_τ^L can be written as $J_\tau^L = J_\tau^C + J_\tau^B$, where

$$J_\tau^C = J(\alpha_{L,\tau}, 0),$$

and

$$J_\tau^B = \int_\tau^\infty e^{-r(s-\tau)} \phi(b, \alpha_{L,s}) ds.$$

So, J_τ^C is the present value of the risk-adjusted instantaneous dividend process accrued to the large shareholder less (plus) the trading costs (gains), and J_τ^B is the present value of the private benefits flow. I define J_τ^C as the marginal utility of the large shareholder. Then, note that without private benefits the actual choice $\alpha_{L,\tau}$ is not optimal, that is

$$J_\tau^C < J(\alpha_{m,\tau}, 0),$$

where $\alpha_{m,\tau}$ denotes the number of shares that the large shareholder would choose as optimal solution in the absence of private benefits, thus behaving as a marginal investor with perfect information. $\alpha_{m,\tau}$ solves the utility maximization problem of the large shareholder when $b = 0$, for a given dynamics of μ_t and $\bar{\mu}_t$, risk aversion coefficient a^L , dividends shock volatility σ_D , initial updating weight $\bar{g}(0)$, and using the fact that in the absence of private benefits $\sigma_\epsilon^2 = 0$. Therefore, it is possible to compute both J_τ^C and $J(\alpha_{m,\tau}, 0)$, by using the implied dynamics of μ_t , and the parameters estimates.

The gain in terms of private benefits must be at least equal to the loss in terms of marginal utility, choosing $\alpha_{L,\tau}$ rather than $\alpha_{m,\tau}$:

$$\phi(\alpha(L, t)) - \phi(\alpha(m, t)) > J(\alpha_{m,t}, 0) - J_\tau^C,$$

from which I derive the lower bound for b , where $\phi(\alpha(L, t)) = b * \alpha_j(L, t)$ and $\phi(\alpha(m, t)) = b * \alpha_j(m, t)$, and $\alpha_j(L, t)$ and $\alpha_j(m, t)$ are the thresholds associated to $\alpha_{L,t}$ and $\alpha_{m,t}$, respectively. So,

$$b > \frac{J(\alpha_{m,t}, 0) - J_\tau^C}{(\alpha_j(L, t) - \alpha_j(m, t))} = b^l.$$

On the other hand, the private benefits that the large shareholder can extract from the stake are not large enough to make convenient for the large shareholder to increase his stake up to a higher threshold. In other words, jumping to a higher threshold would produce gains in terms of private benefits not sufficient to cover the loss in terms of marginal utility:

$$\phi(\alpha_{j+1}(L, t)) - \phi(\alpha_j(L, t)) < J(\alpha_{j+1,t}, 0) - J_\tau^C,$$

from which I derive the upper bound for b , where $\alpha_{j+1}(L, t)$ is the threshold above the one associated to $\alpha_{L,t}$ (e.g., if $\alpha_{L,t}$ is 53%, then $\alpha_j(L, t)$ is 50% and $\alpha_{j+1}(L, t)$ is 66%). Therefore,

$$b < \frac{J(\alpha_{j+1,t}, 0) - J_\tau^C}{(\alpha_{j+1}(L, t) - \alpha_j(L, t))} = b^u.$$

In table of results in the following section, I report the mid point between lower and upper bounds, that is $(b^l + b^u)/2$.

4.7. Estimating the Price Impact

The estimation of the price impact of the private benefits involves a simple counterfactual analysis. First, note that the observed stock price is the stock price that reflects the private benefits. In fact, the observed stock price, at the discrete dates at which the large shareholder discloses the stake, depends on the large shareholder's stake.

Then, in the counterfactual analysis, I compare the observed stock price, at the discrete disclosure dates, with two unobservable stock prices: (i) the stock price in the absence of private benefits, that is the equilibrium stock price when there is one large shareholder that is fully informed about the true expected dividends payout of the firm, and (ii) the stock price in the absence of a large shareholder, that is the equilibrium stock price when the marginal investors do not receive any additional signal to update their beliefs at the discrete disclosure dates. Let P^m denote the equilibrium stock price in the absence of private benefits, and P^n the equilibrium stock price in the absence of a large shareholder, then

$$P_\tau^m = P(\bar{\mu}(\alpha_{m,\tau}), \rho(\alpha_{m,\tau})),$$

$$P_\tau^n = P(\bar{\mu}_\tau = \bar{\mu}_{t<\tau}, \rho(0)),$$

and

$$\psi^m = \frac{\tilde{P}_\tau - P_\tau^m}{P_\tau^m},$$

$$\psi^n = \frac{\tilde{P}_\tau - P_\tau^n}{P_\tau^n},$$

where ψ^m and ψ^n denote the (percentage) price impact in the two different cases, respectively.

I estimate the price impact at the disclosure discrete dates τ , that is at biannual frequency. I compute both P^m and P^n by using equation (9). As for P_τ^m , I first determine the optimal stake of the large shareholder in the absence of private benefits, $\alpha_{m,\tau}$, for a given dynamics of μ_t and $\bar{\mu}_t$, risk aversion coefficient a^L , dividends shock volatility σ_D , initial updating weight $\bar{g}(0)$, and using the fact that in the absence of private benefits $\sigma_\epsilon^2 = 0$. Then, I derive the equilibrium risk premium and the updated belief on the dividends drift by the marginal investors ($\bar{\mu}_\tau$), thus obtaining the equilibrium stock price. Computing P_τ^n , instead, requires only the dynamics of $\bar{\mu}_t$, and the equilibrium risk premium is simply given by $a^L \sigma_D^2 r$.

Table 3. Parameters Estimates and Goodness of Fit

	Mean	Median	Std	10th pct	90th pct
σ	0.024	0.016	0.028	0.002	0.051
σ_D	0.279	0.210	0.294	0.038	0.639
a_L	2.664	2.137	1.452	1.131	4.999
b	0.001	0.000	0.008	-0.003	0.009
$\bar{g}(0)$	0.137	0.041	0.172	0.000	0.493
σ_ϵ^2	0.584	0.764	0.405	0.000	0.998
	Mean		Median		
	Actual	Fitted	Actual	Fitted	
Stock Price volatility	1.98	2.05	1.22	1.19	
LS Trading volatility	0.029	0.031	0.016	0.014	
Trade (% of shares)	7.73	11.73	3.64	5.46	
	Actual	Fitted			
N of Trades $\geq 1\%$	10.76%	6.63%			

The table reports statistics on the parameters estimates for the final sample of 78 firms. The parameters are: the volatility of shocks to drift (σ), the volatility of shocks to dividends (σ_D), the large shareholder's risk aversion (a^L), the private benefits b , the initial updating weight of the marginal investors' belief $\bar{g}(0)$, and the noise in the large shareholder's trade observation σ_ϵ^2 . The parameters are estimated firm-by-firm, by using maximum likelihood. σ and σ_D are estimated with daily stock prices and annual earnings-per-share, a^L , $\bar{g}(0)$ and σ_ϵ^2 are estimated with biannual stock prices and ownership shares, b is the average between the lower bound b^l and the upper bound b^u . The goodness of fit shows the comparison between actual empirical moments on stock price, large shareholder's stake and trading (averaged across firms), and estimation-implied moments on stock price, large shareholder's stake and trading. Moreover, the bottom line compares the observed number of trades with the estimated-implied number of trades, above 1% of the outstanding shares, as percentage of the total number of observations.

5. Results

5.1. Model Fit

Table III reports parameter estimates (top panel), and goodness of fit in terms of empirical moments on stock prices and large shareholders' stake and trading (bottom panel).

The volatility of shocks to dividends (σ_D) is widely larger than the volatility of shocks to the fundamental value of the firm (σ), proxied by the time-varying drift of the dividends process. We can interpret in the real data the continuous dynamics of the dividends process

of the model as the daily arrival of news and information about the state of the firm.

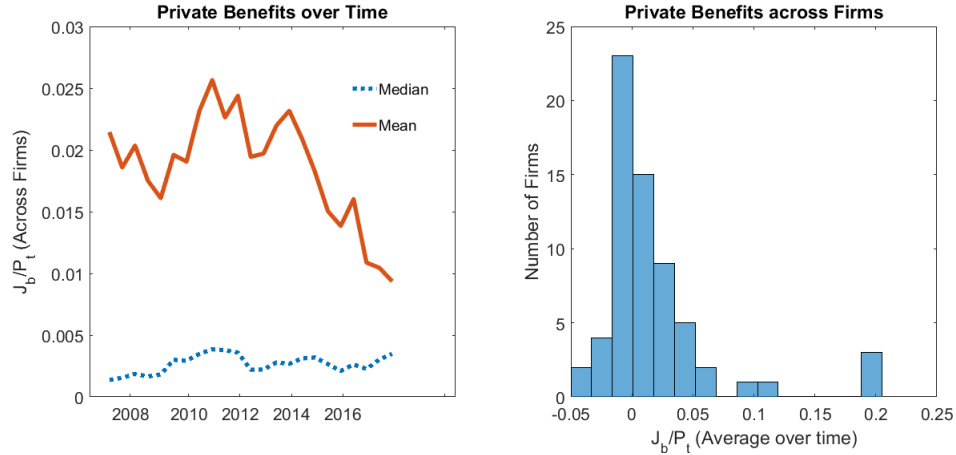
The estimate of the large shareholder's risk aversion coefficient is close to the value calibrated for the aggregate risk aversion. Further, estimates on $\bar{g}(0)$ document heterogeneity across firms in terms of initial weight assigned to the large shareholder's stake by the marginal investors, and also in terms of noise of the information released by the large shareholders with their demand for shares.

The bottom panel compares the model in-sample predictions on the dynamics of stock prices and large shareholders' stake to their corresponding actual values in the data. The estimated model performs well in replicating these features of the data, even though the stylized facts on the dynamics of large shareholders' stake and trading have not been explicitly targeted. Mean and median across firms of the large shareholders' trading volatility are almost equal between real data (1.98%, and 1.22%, respectively) and model-implied estimates (2.05%, and 1.19%), and close in terms of size of trade (7.73%, and 3.64% in real data, and 11.73% and 5.46% in the model). The differences between real data and model predictions, in terms of size of trade, and also in terms of number of trades (10.76% and 6.63%, respectively), are likely due to the exogenous thresholds imposed in model, so that the model tends to predict trades of larger blocks compared to the actual trades, but with a lower frequency.

5.2. *Private Benefits*

I use the estimated parameter b , and the actual stake of the large shareholder, to compute the present value per share of the private benefits flow, defined in the model as J_b , for each firm, and each disclosure date τ . I report results on J_b in terms of stock price, then $J_{b,\tau}/P_\tau$ is the measure of the present value of the private benefits flow extracted by large shareholders with their stake, in terms of equity value of the firm. The left panel of Figure 5 reports the time series of mean and median across firms at each biannual date, and the right panel of Figure 5 reports the distribution of the average over time for each sample firm. Statistics on

Figure 5. Private Benefits over Stock Price



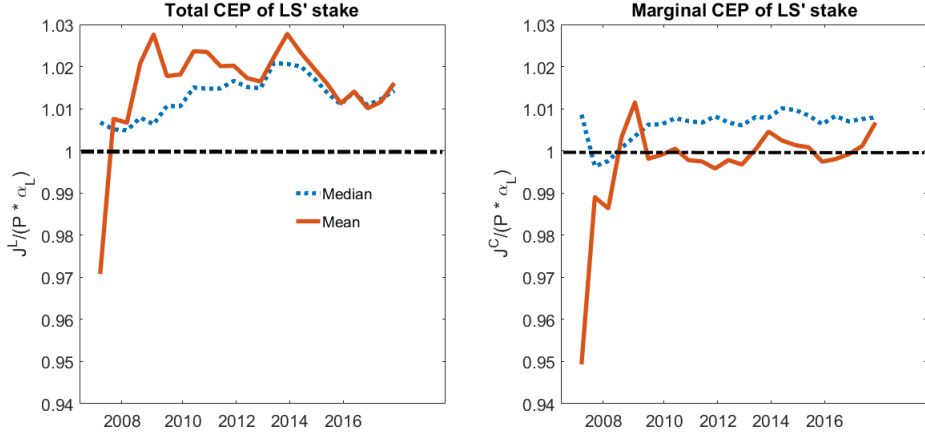
The left panel shows the mean (red line) and the median (blue dotted line) across firms, for each biannual date between March 2004 and September 2016, of the present value of the private benefits flow (J_b), divided by the stock price at the same date. The right panel shows the distribution of the present value of the private benefits flow divided by the stock price over the 78 sample firms, where for each firm I compute the average ratio over time.

the distribution are shown in Table 4.

Private benefits amount to approximately 2% of equity value on average. This number is slightly lower than the estimate of Albuquerque and Schroth (2010). Similarly to Albuquerque and Schroth (2010), I also document a pronounced positive skewness in the distribution across firms, where the mean is much higher than the median, and does not provide an accurate picture of the results. Half of the large shareholders (40% in Albuquerque and Schroth (2010)) extract private benefits less than 1% of the total equity value, and the maximum rate of private benefits is 20% (15%).

I compute the certainty equivalent payoff of the large shareholders' stake, defined in the model as J^L , that is the sum of the present value of the private benefits flow plus the marginal utility, defined as the present value of the risk adjusted dividends flow plus (minus) the trading costs (gains). Remind that the certainty equivalent payoff is simply the valuation of the stake by an investor, so the maximum price at which the investor is willing

Figure 6. Certainty Equivalent of Large Shareholders



The left panel shows the mean (red line) and the median (blue dotted line) across firms, for each biannual date between March 2004 and September 2016, of the certainty equivalent payoff of the largest shareholder (J^L), divided by the stock price at the same date, and the largest shareholder's stake. The right panel shows the mean (red line) and the median (blue dotted line) across firms, for each biannual date between March 2004 and September 2016, of the marginal component of the certainty equivalent payoff of the largest shareholder (J^C), divided by the stock price at the same date, and the largest shareholder's stake.

to buy that block of shares. I report results in terms of stock price, dividing by the large shareholders' stake, so that $J^L / (P_\tau * \alpha_{L,\tau})$ is equal to 1 when the large shareholder values one share exactly as the market does. The left panel of Figure 6 reports the time series of mean and median across firms at each biannual date for $J^L / (P_\tau * \alpha_{L,\tau})$, and the right panel of Figure 6 reports the time series of mean and median across firms at each biannual date for $J^C / (P_\tau * \alpha_{L,\tau})$, that is the marginal component of the certainty equivalent payoff of the large shareholders' stake. Statistics on the distributions of the average over time for each firm are shown in Table 4.

Intuitively, large shareholders value their stakes more than the market price. With risk aversion, an investor is willing to buy a stock only if the certainty equivalent is above the market price of the stock. The difference between the total and the marginal large shareholders' certainty equivalent is due to the private benefits. So, private benefits contribute

Figure 7. Price impact over time. All Firms



The left panel shows the mean (red line) and the median (blue dotted line) across firms, for each biannual date between March 2004 and September 2016, of the price impact of the private benefits extracted by the largest shareholder (ψ^n). The right panel shows the mean (red line) and the median (blue dotted line) across firms, for each biannual date between March 2004 and September 2016, of the price impact of the overall large shareholder’s stake (ψ^m).

to compensate large shareholders for holding very undiversified portfolios, thus keeping the stake valuation of the large shareholders always above the market price of the stock.

5.3. Price Impact

Finally, I use the model estimates to compute the (percentage) price impact of the private benefits of control, and of the overall large shareholder’s stake, defined as ψ^m and ψ^n , respectively. Figure 7 reports the time series of mean and median across all sample firms at each biannual date, and Figure 8 reports the time series of the average across two different types of large shareholders at each biannual date: corporations and individuals. Statistics on the distributions of the average over time for each firm are shown in Table 4.

In general, private benefits of control have positive impact on stock prices, so they are not extracted with cost for the rest of the company shareholders. The average price impact over time fluctuates between 1% and 3%, and the mean is always significantly larger than the median, signalling again highly positive skewness. The price impact of private benefits,

Table 4. Private Benefits and Stock Price

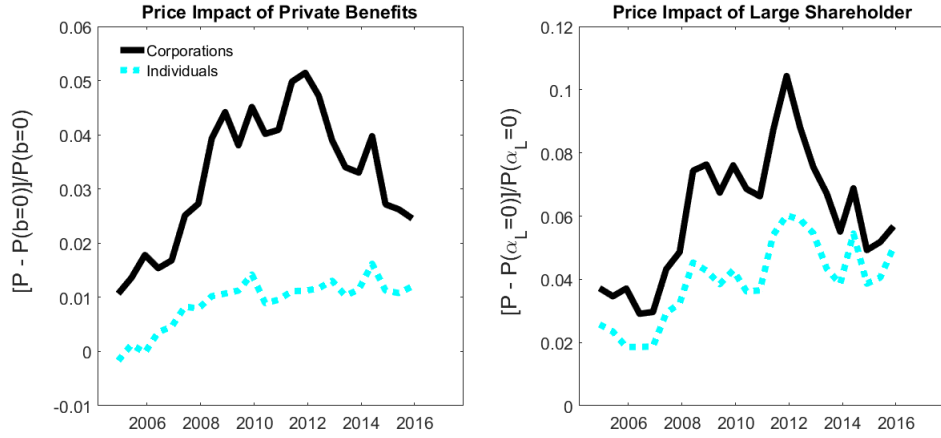
Panel A: All Sample				
	Mean	Median	10th pct	90th pct
J^L/P	1.015	1.014	0.984	1.053
J_b/P	0.018	0.003	-0.015	0.054
J_C/P	0.997	1.006	0.945	1.054
ψ^m	0.020	0.002	-0.021	0.096
ψ^n	0.048	0.028	-0.001	0.128
Panel B: Corporations				
	Mean	Median	10th pct	90th pct
J^L/P	1.018	1.022	0.982	1.063
J_b/P	0.029	0.009	-0.016	0.105
J_C/P	0.989	1.002	0.865	1.052
ψ^m	0.032	0.003	-0.018	0.127
ψ^n	0.060	0.046	0.001	0.149
Panel C: Individuals				
	Mean	Median	10th pct	90th pct
J^L/P	1.013	1.014	1.001	1.023
J_b/P	0.011	0.001	-0.010	0.011
J_C/P	1.002	1.007	0.967	1.059
ψ^m	0.009	0.002	-0.026	0.029
ψ^n	0.039	0.021	-0.002	0.132

The table reports statistics on the total certainty equivalent payoff of the large shareholder (J^L), the marginal certainty equivalent payoff of the large shareholder (J^C), and the present value of the private benefits flow (J^b), divided by the stock price at the disclosure date τ , and on the price impact of private benefits (ψ^m) and the total price impact of the large shareholder's stake (ψ^n). First, I compute the average over time for each firm, then I report in the table mean, median, and 80% confidence interval across firms, for all sample firms, and by type of large shareholder.

in fact, is quite heterogenous across firms. For 15% of the firms private benefits have a negative impact greater than 1%, and for the same proportion of firms private benefits have a positive impact larger than 5%.

The impact of the large shareholder's stake, overall, is even more beneficial to the rest of the investors. For 15% of the firms, in fact, the presence of the large shareholder increases the stock price by more than 10%, and for only 6% of the firms the negative impact is

Figure 8. Price impact over time. Types of shareholders



The left panel shows the mean, across the firms where the largest shareholder is a corporation (black line) and the firms where the largest shareholder is an individual (sky-blue line), for each biannual date between March 2004 and September 2016, of the price impact of the private benefits extracted by the largest shareholder (ψ^n). The right panel shows the mean, across the firms where the largest shareholder is a corporation (black line) and the firms where the largest shareholder is an individual (dotted line), for each biannual date between March 2004 and September 2016, of the price impact of the overall large shareholder's stake (ψ^m).

greater than 1%. In general, the positive price impact of both private benefits and large shareholder's stake is substantially greater during the crisis of 2011-2012. This suggests that large shareholders support stock prices and are significantly beneficial to the rest of the investors over negative economic cycles. In untabulated results, I find that the average price impact of private benefits is 3.17% at the beginning of 2012 (at the boom of the European sovereign debt crisis) and only 0.96% at the beginning of 2007, before the start of the great financial crisis. Moreover, the average price impact of large shareholders, overall, is 7.87% at the beginning of 2012, and only 2.27% at the beginning of 2007.

Table 4 shows that corporate large shareholders exert more beneficial effect to the stock price than individuals, both in terms of private benefits and presence of large shareholder, and both over time and across firms. The combination between positive price impact of corporate large shareholders and positive estimate of private benefits extracted by corporate

large shareholders offers the evidence of a synergistic effect between owner and owned firm. In this case, in fact, I document a reciprocal beneficial effect so that private benefits are not extracted at cost for the rest of shareholders, and they can be properly defined as synergies.

6. Conclusion

The paper estimates private benefits of control using the restrictions provided by a dynamic model of optimal shareholding, with asymmetric information and heterogeneous shareholders. The model equilibrium conditions allows to quantify the price impact of large shareholders and private benefits of control over time. The estimation results provide evidence that large shareholders have positive impact on stock prices that does not vanish over time, so that they extract private benefits without cost for the rest of the company shareholders. On the other side, private benefits contribute to compensate large shareholders for holding very undiversified portfolios. When the large shareholder is a corporation, for instance, this reciprocal beneficial effect sheds light on a synergy between owned and owner firm.

While this paper proposes the first structural approach to quantify the price impact of large shareholders over time, both the theoretical and the empirical analysis is far from being exhausted. The main challenge for future research is to disentangle the two drivers of the heterogeneity between large shareholder and marginal investors, namely the opportunity to extract private benefits and the superior information.

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Appendix A. Proofs

Appendix A.1. Investors' Optimality Conditions

To derive the optimality conditions of the marginal investor, I follow the same approach of DeMarzo and Urosevic (2006). First, I formulate the investor's conjecture about the price process, and I conjecture a value function for the investor in the customary form as in DeMarzo and Urosevic (2006). Then, I derive the Bellman equation, to be maximized with respect to the control variables α and c . Finally, I derive the equilibrium risk premium, and the equilibrium share price in Proposition 1.

In a CARA-utility framework, with normality assumption on the dividend payout, the conjecture of the price process is the following

$$dP_t = (rP_t + \rho_t - \bar{\mu}_t)dt,$$

that is the share price grows at the riskless rate, plus a risk premium component ρ to compensate the investor's risk aversion, and to be determined in equilibrium, minus the biased expected dividend payout.

Then, for a given price process, the investor's optimality conditions are the followings:

$$u_c = J_W,$$

where u_c denotes the marginal utility from consumption, and J_W is the partial derivative of the value function of the investor, that is his expected payoff on each point in time, with respect to the state variable wealth, and

$$\alpha_{i,t} = \frac{\mu_{i,t} - \bar{\mu}_t + \rho}{a r \sigma^2},$$

where $\bar{\mu}_t$ is the average expected dividend across the marginal investors, and the risk premium ρ compensates the investor's risk aversion. By market clearing, that is $\int_i \alpha_{i,t} di =$

$1 - \alpha_{L,t}$ for each t , the equilibrium risk premium is given by

$$\rho_t = (1 - \alpha_{L,t})a^I r \sigma^2,$$

where $\alpha_{L,t}$ is the stake held by the large shareholder at time t , and a^I is the aggregate risk aversion coefficient

$$\frac{1}{a^I} = \int_i \frac{1}{a^i} di$$

In fact, let the riskless holdings of the investor to evolve as follows, for given consumption level c_t , shareholding α_t , and share price P_t

$$dB_t = (rB_t - c_t)dt + \alpha_t dD_t - P_t d\alpha_t,$$

The wealth of the investor is defined as $W = \alpha P + B$. Then, the expected value of the wealth accumulation over time, $dW_t = dB_t + \alpha_t dP_t + P_t d\alpha_t$, is

$$\frac{1}{dt} E_t[dW] = rW_t + \alpha_t(\mu_{i,t} + \rho_t - \bar{\mu}_t) - c_t = rW_t + \alpha_t(\rho_t + e_{i,t} - \bar{e}_t) - c_t$$

Consider the value function

$$J(W, t) = \frac{1}{r} u \left(r[W + y_t] + \frac{(R - r)}{ar} \right),$$

where y_t is the certainty equivalent of the investor at time t

$$y_t = \int_t^\infty e^{-r(s-t)} \left(\frac{1}{2} \alpha_t^2 r \sigma^2 a \right) ds$$

The value function $J(W, t)$ satisfies the following HJB equation

$$\max_{\alpha, c} J_t + J_W dW + \frac{1}{2} J_{WW} dW^2 + u(c) = RJ,$$

Substituting dW , taking the expectation at t , and maximizing over c and α , I obtain the equations of optimality in the desired form, where $J_W = arJ$, and $J_{WW} = -aJ_W$.⁷□

Appendix A.2. Equilibrium Stock Price

Consider the i -th investor's certainty equivalent at time t for the stake $\alpha_{i,t}$,

$$\int_t^\infty e^{-r(s-t)} \left(\alpha_{i,t} \mu_{i,t} - \frac{1}{2} \alpha_{i,t}^2 a^i \sigma_D^2 r \right) ds,$$

that, with constant interest rate r , can be written as

$$\frac{\alpha_{i,t} \mu_{i,t} - \frac{1}{2} \alpha_{i,t}^2 a^i \sigma_D^2 r}{r},$$

that in equilibrium must be equal to the cost of the stake, that is

$$\frac{\alpha_{i,t} \mu_{i,t} - \frac{1}{2} \alpha_{i,t}^2 a^i \sigma_D^2 r}{r} = \alpha_{i,t} P_t$$

Then, take the derivative of both sides with respect to α , and solve for α

$$\alpha_{i,t} = \frac{\mu_{i,t} - r P_t}{a r \sigma^2}.$$

Then, impose the market clearing condition, so that $\int_i \alpha_{i,t} = 1(1 - \alpha_{L,t})$, substitute $\alpha_{i,t}$ and solve for P_t ,

$$P_t = \frac{\bar{\mu}_t - (1 - \alpha_{L,t}) * a^I \sigma_D^2 r}{r},$$

where $\bar{\mu}_t = \int_i \mu_{i,t} di$, and $a^I = \int_i (1/a^i) di$. So, P_t can be written as

⁷For the proof that the Bellman equation holds, and the conditions to avoid doubling strategies and Ponzi scheme, the reader can refer to DeMarzo and Urosecic (2006).

$$P_t = \int_t^\infty e^{-r(s-t)}(\bar{\mu}_s - \rho_s)ds$$

where $\rho_s = (1 - \alpha_{L,t}) * a^I \sigma_D^2 r$

Appendix A.3. Evolution of average expected dividend

First, I derive the evolution over time of the demand for shares of the investors, by setting $\alpha_{i,t}$ as function of $\mu_{i,t}$ and $\bar{\mu}_t$, then using Ito's lemma. The investor, then, buys (sells) company shares when the change in her expectation about the future dividend is greater (lower) than the change in the average expectation about the future dividend. In fact, setting $\alpha_{i,t} = f(\mu_{i,t}, \bar{\mu}_t)$, then

$$d\alpha_{i,t} = \frac{1}{ar\sigma^2}(d\mu_{i,t} - d\bar{\mu}_t),$$

where $d\mu_{i,t} = k_i(dDt - E_{i,t}(dD_t))$.

Note then that the market clearing condition holds also in a dynamic fashion: $\int_i d\alpha_{i,t} = 0$, so that

$$\int_i d\mu_{i,t} di = M d\mu_t$$

where M is the measure of the marginal investors. It follows that

$$d\mu_t = \frac{1}{M} \int_i (k_i(dDt - E_{i,t}(dD_t))) = \bar{k}(dDt - \mu_t),$$

since k_i and $E_{i,t}(dD_t)$ are independent.

Appendix A.3.1. Proposition 1

The proof of the proposition 1 is simply the combination between the two previous proofs, at the disclosure dates τ . When the large shareholder discloses his stake, then each investor

i revises the prior on μ_t , and so the optimal demand for shares, so that, applying again Ito's lemma,

$$d\alpha_{i,\tau} = \frac{1}{ar\sigma^2}(d\mu_{i,\tau} - d\bar{\mu}_\tau),$$

where $d\mu_{i,\tau} = g_i(\tau^-)(\alpha_{L,\tau} - E_{t<\tau}(\alpha_{L,\tau}))$, with $E_{t<\tau}(\alpha_{L,\tau})$ equal across the investors as it is common knowledge at τ , and

$$d\bar{\mu}_\tau = \bar{\mu}_\tau - \bar{\mu}_{t<\tau}$$

Again, the market clearing condition holds in a dynamic fashion: $\int_i d\alpha_{i,\tau} = 0$, so that

$$\int_i d\mu_{i,\tau} di = Md\bar{\mu}_\tau$$

It follows that

$$d\bar{\mu}_\tau = \frac{1}{M} \int_i (g_i(\alpha_{L,\tau} - E_{t<\tau}(\alpha_{L,\tau}))) di = \bar{g}(\tau^-)(\alpha_{L,\tau} - E_{t<\tau}(\alpha_{L,\tau}))$$

Then, consider the i -th investor's certainty equivalent at time τ for the stake $\alpha_{i,\tau}$,

$$\frac{\alpha_{i,\tau}\mu_{i,\tau} - \frac{1}{2}\alpha_{i,\tau}^2 a^i \sigma_D^2 r}{r},$$

that in equilibrium must be equal to the cost of the stake, that is

$$\frac{\alpha_{i,\tau}\mu_{i,\tau} - \frac{1}{2}\alpha_{i,\tau}^2 a^i \sigma_D^2 r}{r} = \alpha_{i,\tau} P_\tau$$

Then, take the derivative of both sides with respect to α , and solve for α

$$\alpha_{i,\tau} = \frac{\mu_{i,\tau} - rP_\tau}{ar\sigma^2}.$$

Then, impose the market clearing condition, so that $\int_i \alpha_{i,\tau} = (1 - \alpha_{L,\tau})$, substitute $\alpha_{i,\tau}$ and solve for P_τ ,

$$P_\tau = \frac{\bar{\mu}_\tau - (1 - \alpha_{L,\tau}) * a^I \sigma_D^2 r}{r},$$

where $\bar{\mu}_\tau = \int_i \mu_{i,\tau} di$, and $a^I = \int_i (1/a^i) di$. So, P_τ can be written as

$$P_\tau = \int_\tau^\infty e^{-r(s-\tau)} (\bar{\mu}_s - \rho_s) ds$$

where the equilibrium risk premium is $\rho_\tau = (1 - \alpha_{L,\tau}) * a^I \sigma_D^2 r$

Appendix A.4. Proposition 2

Next, consider the large shareholder's certainty equivalent payoff,

$$V(\alpha_{L,\tau}) - (\alpha_{L,\tau} - \alpha_{L,\tau^-}) P_\tau,$$

write P_τ and V_τ in the explicit form, take the derivative with respect to α , and solve explicitly for $\alpha_{L,\tau}$,

$$\alpha_{L,\tau} = \frac{\mu_t - \bar{\mu}_\tau + (1 + \alpha_{L,\tau^-}) a^I \sigma_D^2 r + \phi(\alpha_{L,\tau})}{\Delta},$$

where $\Delta = 2a^I \sigma_D^2 r + a^L \sigma_D^2 r$.

Now, let define

$$\alpha_{L,\tau}(\bar{\mu}_{t < \tau}) = \frac{(1 + \alpha_{L,\tau^-}) a^I \sigma_D^2 r}{\Delta},$$

then

$$\alpha_{L,\tau} = \frac{\mu_t - \bar{\mu}_\tau + \phi(\alpha_{L,\tau})}{\Delta} + \alpha_{L,\tau}(\bar{\mu}_{t < \tau})$$

then substitute $\bar{\mu}_\tau = \bar{\mu}_{t<\tau} + \bar{g}(\tau^-)(\alpha_{L,\tau} - \alpha_{L,\tau}(\bar{\mu}_{t<\tau}))$, collect common terms over $\alpha_{L,\tau}$ and $\alpha_{L,\tau}(\bar{\mu}_{t<\tau})$, and solve again explicitly for $\alpha_{L,\tau}$, thus obtaining the desired form.

Appendix B. Kalman Filter

The Kalman filter allows to reconstruct the dynamics of a latent variable, by using observable variables, and a given known relationship between latent and observable variables. The relationship between observed and unobserved variables forms the *measurement equation*, while the evolution over time of the latent variable is called *transition equation*.

In few words, the filter starts from a prior on the latent variable, and forms a prediction of the next step value of the latent following the diffusion described in the transition equation. Then, the filter makes a forecast of the observable variable based on the prediction of the latent, by using the relationship described in the measurement equation. At each time step, the filter generates an error, given by the distance between the actual value of the observable and the forecast. The error is then used to update the prior on the latent, for a given weight assigned to the error, which is called *Kalman Gain*. Moreover, the errors depend on the model parameters, so under the assumption of normality the errors are used to construct a likelihood function that is maximized with respect to the model parameters.

In my set up, the transition equations describe the evolution of the average expected dividend across the marginal investors, and the true time-varying dividends drift, that are defined in the equation (7) and (8), respectively. The parameter \bar{k} in equation (7) is allowed to vary over time

$$k_t = \frac{\nu_t}{\nu_t + \sigma_D^2},$$

where $\nu_t = w_t + \sigma$, and at each time step w_t is updated by using $(1 - k_{t-1})\nu_{t-1}$, and initializing the recursion with a large value of ν_0 . This procedure allows to proxy the prior update on the dividends drift across the marginal investors. Moreover, the state prediction on $\bar{\mu}_t$ has

variance equal to $\omega_t + k_t^2 \sigma_D^2$, where the second term is the variance of the state, conditional at time t , derived by the market clearing equation. The state prediction on μ_t has variance equal to $\omega_t + \sigma^2$, ω_t is updated at each time step according to the Kalman Gain, and the recursion is initialized with a large value of ω_0 .

The measurement processes, instead, come from the pricing equation defined in (9), and I assume that the actual prices are observed with noise, for instance due to microstructure issues, so that

$$\tilde{P}_t = P_t + \epsilon_{1,t},$$

where the noises are gaussian, with zero mean and variance σ_P^2 .

Hence, starting from an initial guess $\{\mu_0, \bar{\mu}_0\}$, I define the prediction for $\{\mu_1, \bar{\mu}_1\}$ by using the transition equations. Then, given the prediction on the state, I compute the forecast for the share price P_1 , thus obtaining a prediction error, by using the actual observations \tilde{P}_1 . Combining the errors with the Kalman Gain, I update the prior for the state, and iterate recursively the filter up to the end of the time series. The Kalman Gain is the optimal weight to assign to prediction error in order to revise the prior on the state variable. It is derived by minimizing the conditional variance (covariance matrix) of the state variable(s).

Appendix C. Simulation Study

To test the accuracy of the estimation methodology, I perform a numerical analysis over 1000 simulations. I simulate the dynamics of the dividends time varying drift, and the dividends process of the firm, according to the equations (1) and (2), with daily frequency ($\delta t = 1/250$), for a set of arbitrary values of σ and σ_D . I also simulate a mass of marginal CARA-maximizer investors, who observe dD_t and update their prior on μ_t according to their heterogenous prior variances, thus obtaining an average expected dividend across the marginal investors, $\bar{\mu}_t$, for each t .

Table C.5. Parameters Estimates: Simulation Study

	True	Mean	Median	10th pct	90th pct
σ	0.1	0.093	0.093	0.087	0.099
σ_D	0.2	0.202	0.199	0.132	0.274
a_L	8	8.71	7.97	7.10	10.36
b	0.02	0.019	0.021	0.007	0.027
$\bar{g}(0)$	0.1	0.097	0.099	0.082	0.105
σ_ϵ^2	0.2	0.189	0.188	0.138	0.245

Next, for given parameter b and thresholds of the private benefits function, aggregate risk aversion coefficient a^I , large shareholder's risk aversion coefficient a^L , initial average weight $\bar{g}(0)$ and noise σ_ϵ assigned to the large shareholder's choice by the marginal investors for updating their beliefs at the discrete disclosure dates, I obtain the time series of the large shareholder's stake and the equilibrium stock price with biannual frequency ($\delta\tau = 0.5$).

Then, using daily stock prices, I estimate σ and σ_D , and I infer the dynamics of μ_t and $\bar{\mu}_t$, which I use in the second step to estimate the remaining parameters by using biannual stock prices and large shareholder's stakes. Finally, I identify the parameter b following the identification approach described above. Table V reports mean, median, and 80% confidence interval, over 1000 simulations, for the six parameters against the true arbitrary value of the parameters.